An aid agency is packaging bags of seeds for distribution in a community where farmers have been unable to save enough seeds to plant crops this year. Rather than just giving them food, the agency wants to give each farmer a 10 pound bag of seed. The bags are filled automatically by a machine. Suppose that the actual weight of a randomly chosen bag varies according to a normal distribution with a mean of 10.2 lbs and a standard deviation of .5 lbs.

1. What is the probability that a randomly chosen bag will be less than the 10 lbs that they hope to be giving each farmer?

$$P(X < 10) = P\left(Z < \frac{10 - 10.2}{0.5}\right) = P(Z < -0.40) = .5 - .1554 = .3446$$

2. What is the probability that a randomly chosen bag will actually weigh more than 10.5 lbs?

$$P(X > 10.5) = P\left(Z > \frac{10.5 - 10.2}{0.5}\right) = P(Z > 0.60) = .5 - .2257 = .2743$$

3. What is the probability that a randomly chosen bag will contain between 10 and 10.5 lbs?

$$P(10 \leq X \leq 10.5) = P\left(-0.40 \leq Z \leq 0.60\right) = .2257 + .1554 = .3811$$

4. Since the agency cannot really alter the machine that they are using, they want to relabel the bags such that only 5% of all bags would weigh less than the labeled value. What value should they use on the label?

$$\text{Bottom 5\%: } .5 - .05 = .4500 \Rightarrow Z = -1.645 = \frac{X - 10.2}{0.5} \Rightarrow X = 9.3775 \text{ lbs.}$$